

## PENDULUM LENGTHS AND PERIODS

### A POWER FUNCTION

A group of students is attempting to determine the mathematical relationship between the length of a pendulum and its period (time required for one complete swing of a pendulum). The group makes a simple pendulum from string and washers and then suspends it from the ceiling. They record the pendulum's period for each of 12 string lengths.

Sample data:

Length (cm)	Time (sec)
6.5	0.51
11.0	0.68
13.2	0.73
15.0	0.79
18.0	0.88
23.1	0.99
24.4	1.01
26.6	1.08
30.5	1.13
34.3	1.26
37.6	1.28
41.5	1.32

1. Make a scatter plot of the data set on the graphing calculator.
2. At first glance, the data seems to be linear. Use **LinReg** to find the equation of the "line of best fit". What is the equation you obtained? Graph the line by entering the equation in the  $\boxed{Y=}$  table.
3. Notice however that the line does not "fit" the data very well. We recall now that the function relating the length  $L$  of a pendulum and the time  $T$  for one period is the following:

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{where } g = 981 \text{ cm/sec}^2, \text{ the gravitational constant.}$$

This is not a linear relationship, but instead a power function which can be rewritten as follows:

$$T = 2\pi\sqrt{\frac{1}{g}} \sqrt{L} = a\sqrt{L} = aL^{0.5} \quad \text{where } a \text{ is a constant, and } T \text{ and } L \text{ are variables}$$

Since the relationship between a pendulum's length and its period is of the form  $y = a(x)^b$ , we need to find the function of best fit by choosing **PwrReg** under  $\boxed{\text{STAT}}$  **CALC** on the graphing calculator.

4. What is the power function you obtained for the data set?
5. To check our function against the official function listed above, we can show that the value of  $2\pi\sqrt{\frac{1}{g}}$  is about 0.2006. Compare this value to the value of  $a$  in your power function. Are they reasonably close?

6. According to the function found in part 4, what is the time required to complete one full swing of the pendulum if the length is 10 cm?
7. How long must a pendulum be in order to have a 2 second period?

This example is adapted from the TI83 Graphing Calculator Guidebook, p. 12-2.