# Advanced Mathematics in the Grade 8 to High School Classroom

***Course Description:***

The purpose of the Advanced Mathematics Institute will be to prepare educators to work with a target population of advanced learners in the field of mathematics. Participants will develop their own portfolio of lesson plans, instructional materials, and resources that they can use in their classrooms. Each of the lessons and related materials in the portfolio will reflect the specific mathematical theme involving a branch of mathematics, on which that day of the institute will focus. The focus of these materials will be the use of reality-based application problems.

Participants will be encouraged to have the materials in their portfolios reflect a variety of teaching and assessment models, so as to demonstrate knowledge of a variety of pedagogical techniques. The use of student-guided explorations, small group activities, as well as stressing the role of technology in the classroom will be implemented. In addition, with each lesson, participants will design assessment items, including open-response questions, to use with their students. These types of assessment items will be modeled through the use of a mathematics journal that participants will regularly respond to as part of the institute.

***Course Delivery Information:***

The Institute will take place on Tuesdays and Thursdays from 8 AM – 3 PM for three consecutive weeks in July. The dates for Summer 2005 are July 5, 7, 12, 14, 19, 21, 26, and 28.

In addition, four follow-up sessions will be held during the fall. They will take place on Saturday mornings from 8 AM – 1 PM. The dates are September 17, October 1, 15, 29.

All sessions will be held at the Twain School Building located at 20 Front Street in West Springfield.

***Grading Criteria:***

Students will receive a grade for this course based on the percentages indicated in the chart to the right. The grade will consist of six components, including the integration of content, the development of a curriculum unit, one topic presentation to the class by each student, work in a mathematics journal, the writing of a reflection paper at the end of the course, as well as the student's participation and attendance.

| ***Components*** | ***Percentage*** |
|---|---|
| Content Integration | 25% |
| Curriculum Unit | 25% |
| Mathematics Journal | 20% |
| Topic Presentation | 10% |
| Reflection Paper | 10% |
| Participation | 5% |
| Attendance | 5% |

***Content Integration and Topic Presentation:***

Based on the mathematical theme of each session, students will be asked to explore a challenging topic in more depth. They will be asked to outline the important vocabulary, highlight special cases and problematic areas for students, develop problems and/or assignments that demonstrate the conceptual idea, and discuss how they would deliver the content in their own mathematics classes. The instructor will review their ideas for integrating content and provide feedback to the students.

Choosing a topic the students explored through their content integration study, the student will then present it to the class. The entire class will actively participate during the presentation. The presentation will also be reviewed and graded by their peers using a rubric provided by the course instructor. Students are encouraged to choose topics for their content integration assignments that could also be incorporated into the curriculum project described below.

***Mathematics Journal:***

Each class session, students will be given a series of five mathematical writing prompts to respond to before the next session. The writing prompts will contain problems or concepts related to the theme of the day. During the next class session, the prompts will be collected and graded.

Included, as part of the journal, will be one "learning" writing prompt per session that will encourage the students to reflect on the topics, activities, and presentations that took place in the class. This is an opportunity for the students and the teacher to converse outside of the classroom, and expand each other's knowledge of mathematics, learning, and teaching styles.

***Reflection Paper:***

The culmination of the course will be the development of the reflection paper, which should discuss the student's educational philosophy and personal growth during the class. Students will assess their progress, and address any concerns they may have. The one "learning" prompt each week can be used as a resource for this assignment.

***Curriculum Unit (Outside Project)***

Students will be required to develop an entire unit for a course in which they will teach in the future, preferably during the next school year between September 1 and October 28, 2005. This assignment will be assigned during the break between the summer and follow-up sessions for the course. It will be due during the first Saturday session held in September.

The unit should consist of an overview, individual lesson plans for each concept or activity introduced, as well as assessment. An overview, which provides a description where the unit can be inserted into the current curriculum, timeline for the unit, foundational and detailed learning objective, as well as a listing of possible additional resources that can be used. The lesson plans should contain specific learning objectives, listing of prerequisite knowledge, a description of the instructional methods or activities, suggested adaptations for the lesson, and methods of assessment. Finally, a variety of evaluation strategies should be employed so that all aspects of learning can be assessed. We will address student outcomes as indicated by the assessments of these projects.

***Attendance:***

Attendance is a mandated and graded part of this course. It is graded objectively by taking the number of class hours present and dividing by the total number of 65 hours. For example, if someone missed two hours of class one day, and an entire session at some other point, that would mean that they attended a total of 46 hours. This would give the student an 88% attendance average, for 58/65 * 100% = 89%. This would be 10% of the participation/attendance portion of the grade.

I believe it is essential to count attendance as part of the grade to encourage students to attend all class meeting if possible. There will be many things will come up during the year, especially on Saturdays. By having attendance as part of the graded portion of the class, it helps a person prioritize what is important. There are only 13 class sessions scheduled. I t is essential to be present at all class meetings. There are no excused absences.

***Participation:***

Students will be expected to participate actively in class including small group activities. Participation in class requires you to come prepared, with your materials and homework. Feel free to ask questions at any time. I can only help you, if I know in which areas you are having difficulty. I am willing to provide help on an individual basis.

Participation will also be scored objectively by using a rubric. Following each class, I will complete a quick rubric for each student. Up to eight points per class session will be awarded, for a total of 104 points by the end

of the course. First, students can earn points for having papers, journal entries, and presentations ready and submitted on the day they are due. They will also earn participation points for providing constructive written feedback to classmates on presentations, as well as adding to the class discussions and activities each class. This would be the other 10% of the participation and attendance grade.

# Topics and Assignments for Each Session

**Session 1**
July 5th

Theme: **Patterns, Number Sense, and Operations**

Activities:
Discuss Class Syllabus, Assignments, and Expectations
Pre-Assessment of Participant Knowledge Base
Spread of Infectious Diseases; An Introductory Activity
An Exploration of Pi in Everyday Objects
The Binary Number System

Homework:
1. Perform a content integration exploration for a topic related to today's theme. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *Rich Problems – Big Payoffs*

**Session 2**
July 7th

Theme: **Algebra I**

Activities:
Order of Operations and The 24 Game
Laws of Exponents
Factoring 101
Operations on Rational Expressions

Homework:
1. Perform a content integration exploration for a topic related to today's theme. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *Algebra: Real-life Investigations in a Lab Setting*

**Session 3**
July 12th

Theme: **Technology ~ TI Graphing Handheld Technology**

Activities:
The Milling Machine ~ Equations of Lines and Their Domains
Matrix Operations
The CBL, Data Plots, and Statistics
Linear Regression
Student Topic Presentations

Homework:
1. Perform a content integration exploration for a topic related to today's theme. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *A Graphical Approach to the Quadratic Formula.*

**Session 4** Theme: **Algebra II**
July14th Activities: Solving Linear Systems using Matrices
Mathematical Modeling with Real World Data
Linear Programming
Conic Sections
Student Topic Presentations

Homework:
1. Perform a content integration exploration for a topic related to today's theme. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *Turning Students into Problem Solvers*
4. Read: *Using Graphing Calculators to Model Real-World Data*

**Session 5** Theme: **Technology ~ Geometer's Sketchpad and Cabri Jr.**
July 19th Activities: Constructions of Quadrilaterals
The Equation of Circles
Application Problems with Multiple Solutions
Circles and Related Angles
Student Topic Presentations

Homework:
1. Perform a content integration exploration for a topic related to today's theme. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *Interactive Technology and Classic Geometry Problems*

**Session 6** Theme: **Geometry**
July 21st **Activities:** **Geometric Transformations and Tessellations**
Maximizing the Volume of a Box
Similarity
The Family of Quadrilaterals
Student Topic Presentations

Homework:
1. Perform a content integration exploration for a topic related to today's theme. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *Equal Arcs, Triangles, and Probability*
4. Read: *Pick's Theorem: What a Lemon!*

**Session 7** Theme: **Functions**
July 26th Activities: Function Notation and Composite Functions
Graphs of Polynomial Functions and their End Behaviors
Determining Horizontal, Vertical, and Slant Asymptotes
Student Topic Presentations

Homework:
1. Perform a content integration exploration for a topic related to today's theme. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *Becoming Flexible with Functions: Investigating United States Population Growth*

**Session 8** Theme: **More on Functions**
July 28th Activities: The Zeros of a Function
Maximum and Minimums of Functions
Application Problems involving Optimization
Student Topic Presentations

Homework:
1. Work on designing a curriculum unit for a topic of your choice. Take a look at what needs to be covered in the year at your particular school. Assume the unit will be incorporated as part of the current curriculum. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *Teaching the Logistic Function in High School*

**Session 9** Theme: **Trigonometry**
September 17th Activities: Trigonometric Functions and Their Graphs
Trigonometric Graphs and Their Translations ~ An Investigation
The Unit Circle and Exact Trigonometric Values

Homework:
1. Perform a content integration exploration for a topic related to today's theme. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *A Direct Approach to Computing the Sine or Cosine of the Sum of Two Angles*

**Session 10** Theme: **More on Trigonometry**
October 1st Activities: Trigonometric Identities
Trigonometric Inverses
Solving Trigonometric Equations

Homework:
1. Perform a content integration exploration for a topic related to today's theme. Make sure to follow the provided guidelines.
2. Write responses to the writing prompts provided for today's topics in your mathematics journal.
3. Read: *Conditional Probability: Its Place in the Mathematics Curriculum*

**Session 11** Theme: **Probability**
October 15th Activities: Combinations and Permutations
Probabilities using Counting Methods
Probabilities of Compound Events
Conditional Probabilities
Discussion of Student Outcomes based on Curriculum Unit Project

Homework:
1. Perform a content integration exploration for a topic related to this week's theme. Make sure to follow the provided guidelines.
2. Write your reflection paper. It should also incorporate your own educational philosophy and discuss your personal growth during this class. Feel free to use your "learning" journal entries to help you.
2. Read: *Class Activities with Student-Generated Data*

**Session 12** Theme: **Statistics**
October 29th Activities: Data Collection and Analysis
Measures of Central Tendency
Variance and Standard Deviation
Statistical Plots (Box-And-Whisker, Stem-and-Leaf, Histogram)
Discussion of Student Outcomes based on Curriculum Unit Project
Post-Assessment of Participant Knowledge Base
Administration of the DOE participant follow-up survey

(Sample Assignment)

***MATHEMATICS JOURNAL***

Topic: **Algebra II** Assigned: **July 14, 2005**

1a. Describe the difference between the notation $f^{-1}(x)$ and $\left[ f(x)^{-1} \right]$. Create a function to illustrate the difference.

b. Describe the difference in definitions between a function and a relation. Give an example of each using an equation, and sketch the respective graphs for each.

c. Define the vertical and horizontal lines tests and explain what they each determine. Then give an example of each using an equation that does not pass each respective test and sketch each graph.

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2a. Consider the rational function $f(x) = \dfrac{x+2}{x^2+3x+2}$. Explain how to find the zeros (if none exist explain why), domain, range, asymptotes, and holes.

b. Sketch a graph of the function in part a.

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3a. If a graph has symmetry to both the x and y axes, what other symmetry must it have? Can a graph that has symmetry to the x-axis be the graph of a function? Explain in words and supplement with an example and a sketch for each.

b. Why are even and odd functions so appropriately named? Describe their respective symmetries and explain why this makes sense.

c. A function can be named, odd, even, or neither. Explain why a function cannot be both even and odd. Give an example using an equation and a sketch.

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4. Consider the functions $f(x) = \dfrac{1}{1-3x}$ and $g(x) = x^2 + 3$.

a. Explain in complete sentences how to find $f(g(x))$ and how to find the domain of $f(g(x))$.

b. Explain in complete sentences how to find $g(f(x))$ and how to find the domain of $g(f(x))$.

c. Give a mathematical justification for your responses in parts a and b.

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5. During today's session, we used the calculator to look at mathematical modeling with real world data. We discussed regression, and the use of the correlation coefficient to judge the accuracy of the regression. Reflect on this or any other activity from today. What parts of the activity did you enjoy the most? What did you learn that was new to you? Have you used the graphing handhelds to discuss this concept with students? How has it worked for you? Was there anything difficult that you would change? Is there anything you would like to learn more about? Is this something that would be applicable to your classroom?

***Required Readings:***


Craine, T. (1996). A graphical approach to the quadratic formula. *Mathematics Teacher*, 89, 34 – 38 & 44 – 46.

Gutmann, T. (2003). A direct approach to computing the sine or cosine of the sum of two angles. *Mathematics Teacher*, 96, 314 – 318.

Holliday, B. & Duff, L. (2004). Using graphing calculators to model real-world data. *Mathematics Teacher*, 97, 328 – 342.

Jackiw, N. (2001). *The Geometer's Sketchpad: Dynamic Geometry Software for Exploring Mathematics* [Computer program]. Emeryville, CA: Key Curriculum Press. (Version 4, Fall 2001)

Jones, G., Thornton, C., McGehe, C. & Colba, D. (1995). Rich problems – big payoffs. *Mathematics Teaching in the Middle School*, 1, 520 – 521.

McCoy, L. (1997). Algebra: real-life investigations in a lab setting. *Mathematics Teaching in the Middle School*, 2, 220 – 224.

McGehee, J. (1998). Interactive technology and classic geometry problems. *Mathematics Teacher*, 91, 204 – 208.

Russell, R. (2004). Pick's theorem: What a lemon! *Mathematics Teacher*, 97, 352 – 355 .

Shealy, B. (1996). Becoming flexible with functions: Investigating United States population growth. *Mathematics Teacher*, 89, 414 – 418.

Smith, R. (2003). Equal arcs, triangles, and probability. *Mathematics Teacher*, 96, 618 – 621.

Somers, K., Dilendik, J., & Smolansky, B. (1996). Class activities with student-generated data. *Mathematics Teacher*, 89, 105 – 107.

Stephens, G. (2002). Teaching the logistic function in high school. *Mathematics Teacher*, 95, 286 – 294.

Verzoni, K. (1997). Turning students into problem solvers. *Mathematics Teaching in the Middle School*, 3, 102 – 107.

Watson, J. (1995). Conditional probability: Its place in the mathematics curriculum. *Mathematics Teacher*, 88, 12 – 17.


***Optional/Recommended Readings:***


Bannard, D. (2004). *The trigonometry workbook with the TI-83 Plus and the Geometer's Sketchpad*. Andover, MA: Venture Publishing.

Bennett, D. (1995). *Pythagoras plugged in: Proof and problems for The Geometer's Sketchpad*. Berkley, CA: Key Curriculum Press.

Brueningsen, C., Bower, B., Antinone, L., & Brueningsen-Kerner, E. (1999). Real-world math with the CBL system: Activities for the TI-83 and TI-83 Plus. *Explorations*. Dallas, TX: Texas Instruments, Inc.

Carlson, R. & Winter, M. (1993). *Algebra experiments I: Exploring linear functions*. White Plains, NY: Dale Seymour Publications.

Carlson, R. & Winter, M. (1993). *Algebra experiments II: Exploring nonlinear functions*. White Plains, NY: Dale Seymour Publications.

Clements, C., Pantozzi, R. & Steketee, S. (2002). *Exploring calculus with The Geometer's Sketchpad*. Berkley, CA: Key Curriculum Press.

Equals. (1989). *Get it together: Math problems for groups, grades 4-12*. Berkeley, CA: University of California.

Gough, J. & Gough, S. (2000). TI Interactive! math for high school. *Explorations*. Dallas, TX: Texas Instruments, Inc.

King, J. (1996). *Geometry through the circle with The Geometer's Sketchpad.* Berkley, CA: Key Curriculum Press.

Lund, C. & Andersen, Edwin. (1998). *Graphing Calculator Activities: Exploring topics in algebra I and algebra II (revised edition).* Menlo Park, CA: Dale Seymour Publications.

Morgan, L. (1997). Statistics handbook for the TI-83. *Explorations.* Dallas, TX: Texas Instruments, Inc.

Muschla, G. & Muschla, J. (1996). *Hands-on math projects wit real-life applications*. West Nyack, NY: The Center for Applied Research in Education.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Newman-Turner, R. & Goodman, R. (2002). Activities for algebra with the TI-83 Plus. *Explorations*. Dallas, TX: Texas Instruments, Inc.

Scher, D. (1995). *Exploring conic sections with The Geometer's Sketchpad.* Berkley, CA: Key Curriculum Press.

Shaffer, D. (1995). *Exploring trigonometry with The Geometer's Sketchpad*. Berkley, CA: Key Curriculum Press.

Weeks, A. (2001). *Calculus in motion: Dynamic animations of calculus concepts.* [Computer software]. Burbank, CA: Audrey Weeks. (www.calculusinmotion.com)

Wyatt, K., Lawrence, A. & Foletta, G. (1998). *Geometry activities for middle school students with The Geometer's Sketchpad*. Berkley, CA: Key Curriculum Press.